Computing the volume of flow from a hydrograph

The volume of water passing through a stream cross-section, V_Q, over a specified duration, T, can be calculated by computing the area under the curve of a hydrograph, which is a plot of instantaneous streamflow, Q, against time, t (Figure 1). The y-axis of a streamflow hydrograph has units of L^3t^{-1}. The x-axis has units of t. The product of the two is a volume, L^3. Because a hydrograph is curvy line, we approximate the continuous curve by assembling a series of rectangles in discrete timesteps. The volume of flow that occurs between time instants is simply the area of the rectangle bounded the time and flow values. The width of a rectangle is equal to the duration between 2 time instants. The height of a rectangle is equal to the average of the streamflow values at the bounding time instants. If we construct rectangles for each time step in the entire duration of the hydrograph, compute their areas, and then add them up, we get the entire area under the hydrograph, or the volume of water that flowed past the outlet.

If we assign a t,Q pair an "address" according to its position in the time series (e.g. t_1 and Q_1). We can write an equation for V_q as follows:

$$V_Q = (t_2 - t_1)\left(\frac{Q_2+Q_1}{2}\right) + (t_3 - t_2)\left(\frac{Q_3+Q_2}{2}\right) + \cdots + (t_N - t_{N-1})\left(\frac{Q_N+Q_{N-1}}{2}\right) \quad (1)$$

Where N is the last observation in the series. We can generalize this equation by denoting the "address" with the subscript *i* so the each term in equation x is

$$(t_{i+1} - t_i)\left(\frac{Q_{i+1}+Q_i}{2}\right) \quad (2)$$

With this generalization, equation x can be written in summation form.

$$V_Q = \sum_{i=1}^{N-1}(t_{i+1} - t_i)\left(\frac{Q_{i+1}+Q_i}{2}\right) \quad (3)$$

Equation 3 simply means that the operations to the right of the summation symbol are performed for each address in the series from i=1 to i=N-1, and then the results for each address are added up.

A spreadsheet offers a convenient solution approach (Figure 2). If time and discharge pairs in a series occupy sequential rows in a spreadsheet, the subscript i can refer to a spreadsheet row.

Suppose you are interested calculating V_Q for the data in Figure 2. The mathematical operation to the right of the summation sign in equation 3 is computed in each row from row i=1 to row i=(N-1). In our example, N-1 = 9, which occurs in excel row 10.

For example, the volume of water that flowed between 9 and 12 hours (t4 and t5) is equal to the area of the red dashed trapezoid I Figure 1, which is also equal to the area of the orange square. The area of the orange square is equal to the product of the lengths of the top or bottom boundary and the side boundary. The length of the top boundary is equal to the duration between the time instants t4 and t4. The length of the side boundary is the average of Q5 and Q4. In this example,

$$V_4 = (12 - 9)\left(\frac{30+18}{2}\right) = 72 \tag{4}$$

Figure 2 illustrates how equation 4 is implemented in Excel. Equation 3 is ultimately solved in cell D13 by summing cells D2 through D 10.

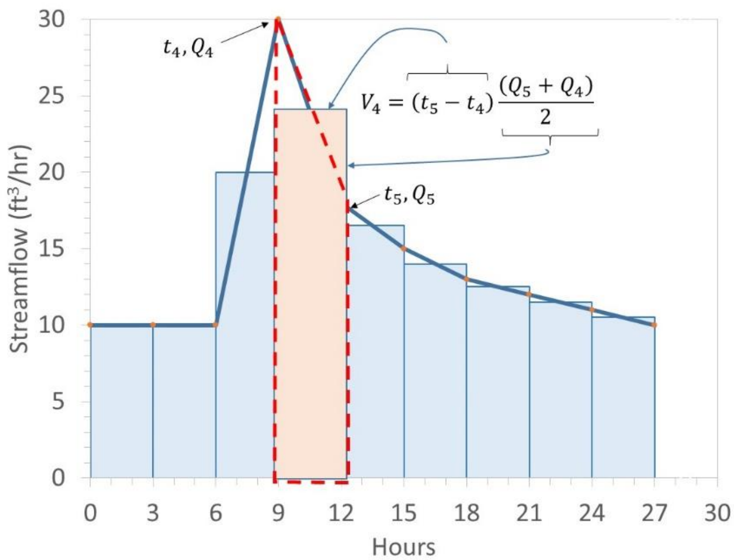
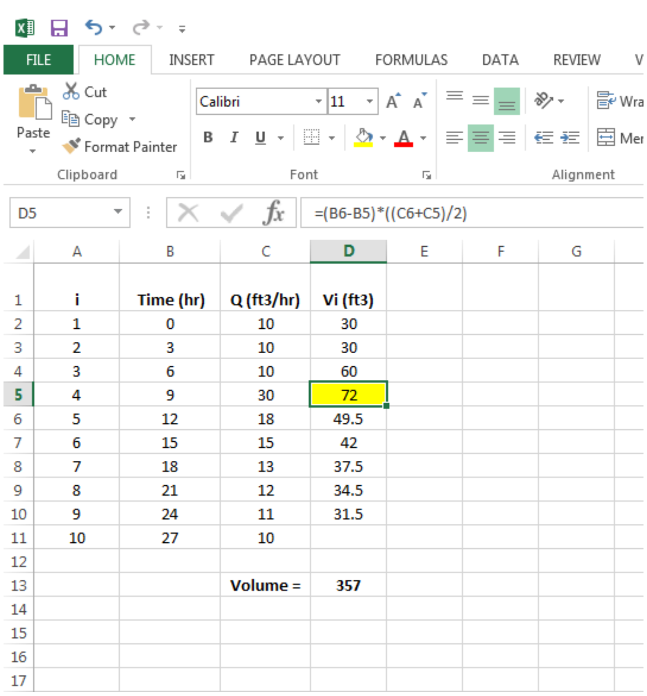


Figure 1. A streamflow hydrograph illustrating how to calculate the volume of flow in discrete timesteps.



i	Time (hr)	Q (ft3/hr)	Vi (ft3)
1	0	10	30
2	3	10	30
3	6	10	60
4	9	30	72
5	12	18	49.5
6	15	15	42
7	18	13	37.5
8	21	12	34.5
9	24	11	31.5
10	27	10	
		Volume =	357

Figure 2. Data and computations to solve Equation 3 using MSExcel.